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# INFLUENCE OF THE MARANGONI EFFECT ON THE STABILITY OF FLOWS OF STRATIFIED LIQUIDS

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A study is made of the effect of a change in interfacial tension on the stability of shear flows of two-component systems. A linear analysis of the stability of motion is performed using the concept of waves of negative energy.

## INTRODUCTION

Shear flows of stratified media take place in the pipeline transport of multicomponent systems. Instability of these flows causes internal waves to be generated in the fluid. Capillary waves at the interfaces play an important role in the generation and propagation of such waves [1, 2].

In the present study, we examine the problem of the generation of capillary waves in an incompressible two-layer liquid with a tangential velocity discontinuity accompanying manifestation of the Marangoni effect — a change in interfacial tension which leads to the motion of adjacent volumes of fluid.

The change in interfacial tension may be due to the presence of surfactants or (in the case of charged systems) the interaction of charges and dipoles in electric double layers, since surface-active ions have a substantial effect on the electrical component of interfacial tension [3]. Since the tension of the "film" of surfactant under the influence of perturbations of the surface is described in a manner wholly analogous to the tension of a "film" of ions, we will limit ourselves to consideration of the effect of surfactants.

As is known, Kelvin—Helmholtz instability occurs during the motion of a two-layer liquid in the region of fairly large wave numbers. Waves with negative energy are present in the region of lower wave numbers, these waves becoming unstable when allowance is made for the factors which deplete them of their energy [4, 5]. Such waves are of particular interest to us, since it is they that promote long-wave instability. The latter phenomenon, in turn, has a serious adverse effect on the efficiency with which stratified liquids can be transported. In physical terms, waves with negative energy are waves for which an increase in amplitude is accompanied by a decrease in the total energy of the system. The concept of negative energy waves was first proposed in studies dealing with electronics and plasma physics [6, 7]. It was later shown that such waves can also be generated in the presence of shear flows in hydrodynamics [4, 5].

In the present investigation, we examine the influence of the Marangoni effect on negative energy waves in a two-layer liquid. It is shown that when such waves are present, the use of surfactants may intensify small disturbances of the interface.

We will examine the motion of two layers of liquid with a sharp interface. The two-dimensional motion of the liquid in a linear approximation is described by the equations

$$\frac{\partial u_j}{\partial t} + u_{0j}\frac{\partial u_j}{\partial x} + \frac{1}{\rho_j}\frac{\partial p_j}{\partial x} = \nu_j \Delta u_j - v_j \frac{\partial u_{0j}}{\partial z}; \qquad (1)$$

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$$\frac{\partial v_j}{\partial t} + u_{0j}\frac{\partial v_j}{\partial x} + \frac{1}{\rho_j}\frac{\partial p_j}{\partial z} + g = \nu_j \Delta v_j;$$

$$\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial z} = 0.$$

Assuming the thickness of the layers of liquid to be large compared to the characteristic wavelength, we will limit ourselves to assigning the conditions on the interface $z = \eta(x, t)$ (in the undisturbed state, $\eta \equiv 0$). We obtain

$$\begin{gathered} v_j = \frac{\partial \eta}{\partial t} + u_{0j}\frac{\partial \eta}{\partial x}; \\ -p_2 + 2\mu_2\frac{\partial v_2}{\partial z} + p_\sigma = -p_1 + 2\mu_1\frac{\partial v_1}{\partial z}; \\ \mu_2\left(\frac{\partial u_2}{\partial z} + \frac{\partial v_2}{\partial x}\right) - p_\tau = \mu_1\left(\frac{\partial u_1}{\partial z} + \frac{\partial v_1}{\partial x}\right). \end{gathered} \tag{2}$$

We will evaluate the effect of the surfactant on the wave motion of the liquid. Let $\Gamma$ be the surface concentration of surfactant at the interface. Then

$$\sigma = \sigma(\Gamma);\ p_\tau = \left(\frac{\partial \sigma}{\partial \Gamma}\right)\frac{\partial \Gamma}{\partial x}.$$

Ignoring surface and bulk diffusion, we write the conservation equation for the surfactant in the form

$$\frac{\partial \Gamma}{\partial t} + \frac{\partial}{\partial x}[\Gamma(u_{0m} + u_m)] = 0.$$

A value of $m = 1$ in the last equation means that the surfactant is soluble in liquid 1; if the surfactant is soluble in liquid 2, then $m = 2$. Writing $\Gamma = \Gamma_0 + \Gamma'$, where $\Gamma_0$ is the concentration at the undeformed interface, when $\Gamma'/\Gamma_0 \ll 1$ we obtain

$$\frac{\partial \Gamma'}{\partial t} + \Gamma_0\frac{\partial u_m}{\partial x} + u_{0m}\frac{\partial \Gamma'}{\partial x} = 0. \tag{3}$$

Assuming that $\mu_1/\mu_2 \ll 1$, we ignore the viscosity of the upper layer of liquid. Since discontinuous flow profiles are allowed at $\mu_1 = 0$, we take the function in the following simple form:

$$u_{0j} = \begin{cases} u_0, & j = 1, \\ 0, & j = 2. \end{cases}$$

The solution of the above-formulated problem has the form [2]:

$$\begin{gathered} u_j = D_j(z)\,e^{i(kx-\omega t)}; \\ v_j = E_j(z)\,e^{i(kx-\omega t)}; \\ p_j = \rho_j A_j (c - u_{0j})\,e^{\mp kz} e^{i(kx-\omega t)}, \end{gathered} \tag{4}$$

where

$$D_j(z) = A_j e^{\mp kz} + \frac{l}{k} B e^{lz} \delta_j;$$

$$E_j(z) = \mp iA_j e^{\mp kz} - iBe^{lz}\delta_j;$$

$$\delta_j = \begin{cases} 0 & \text{at} \quad j = 1; \\ 1 & \text{at} \quad j = 2; \end{cases}$$

$$l^2 = k^2 + \frac{i\omega}{\nu_2}, \quad \operatorname{Re} l > 0.$$

Having set $\Gamma$, $\eta \sim e^{i(kx-\omega t)}$, we can use (1-4) to obtain dispersion relations: at m = 1

$$Z_1 \equiv s\left(\tilde{\omega} - \frac{u_0}{c_0}\right)^2 + \tilde{\omega}^2 + i\varepsilon Y_1\tilde{\omega} - (1 + E) = 0; \tag{5}$$

at m = 2

$$Z_2 \equiv s\left(\tilde{\omega} - \frac{u_0}{c_0}\right)^2 + X_2\tilde{\omega}^2 + i\varepsilon Y_2\tilde{\omega} - 1 = 0, \tag{6}$$

where

$$Y_1 = 2 + \frac{E - i\varepsilon\tilde{\omega}}{\tilde{\omega}^2}\left(\frac{l}{k} - 1\right);$$

$$X_2 = 1 + \frac{E}{\tilde{\omega}^2 + E(l/k - 1)};$$

$$Y_2 = 2 - \frac{2E + i\varepsilon\tilde{\omega}}{\tilde{\omega}^2 + E(l/k - 1)}\left(\frac{l}{k} - 1\right).$$

According to [2], $\varepsilon \leq 10^{-5}/\lambda$, where $\lambda$ is the wavelength, m. Thus, the quantity $\varepsilon$ is small and, in a first approximation, viscosity $\nu_2$ can be ignored. At $\varepsilon \to 0$, (5) and (6) are reduced to the equation

$$Z_0 \equiv s\left(\tilde{\omega} - \frac{u_0}{c_0}\right)^2 + \tilde{\omega}^2 - F = 0, \tag{7}$$

where

$$F = \begin{cases} 1 + E & \text{at} \quad m = 1; \\ 1 & \text{at} \quad m = 2. \end{cases}$$

We can easily use this dispersion equation to obtain the relation

$$\tilde{\omega} = \frac{sr \pm \sqrt{F(s+1) - sr^2}}{s+1}; \qquad r = \frac{u_0}{c_0}, \tag{8}$$

the graph of which is shown in Fig. 1 ($r_1 = \sqrt{F/s}$; $r_2 = \sqrt{F(s+1)/s}$). On the lower branch of the dispersion curve at $0 < r < r_2$

$$\frac{\partial Z_0}{\partial\tilde{\omega}} = 2[(s+1)\tilde{\omega} - sr] < 0,$$

while at $r_1 < r < r_2$ $\tilde{\omega} > 0$.

Since the energy of the waves can be represented in the form [5, 8]

$$W = \omega\frac{\partial Z_0}{\partial\omega}a^2,$$

then at $r_1 < r < r_2$ waves with negative energy exist in the system of reference being examined. The region $r > r_2$ corresponds to Kelvin–Helmholtz instability. As is known [2],

$$c_0 \geqslant c_{\min} \simeq 0.2 \text{ m/sec}$$

In the region of negative energies, $u_0 > r_1 c_0$. Since $r_1 \sim 1$, then waves with negative energy can exist at $u_0 \gtrsim 0.2$ m/sec.

To study the dependence of ω on the wave number k, we rewrite (8) in the form

$$\omega = \frac{su_0 k \pm \sqrt{[(1-s)gk + \alpha_1 k^3](1+s) - su_0^2 k^2}}{s+1},$$

where

$$\alpha_1 = \begin{cases} \alpha_0(1+N) & \text{at} \quad m = 1 \\ \alpha_0 & \text{at} \quad m = 2, \end{cases}$$

or

$$\frac{(s+1)\omega}{u_0 s k_0} = \tilde{k} \pm \frac{1}{\sqrt{s}} \sqrt{\tilde{k} - \tilde{k}^2 + \beta \tilde{k}^3},$$

where

$$\tilde{k} = k/k_0; \quad k_0 = (1-s^2)\, g/u_0^2 s;$$
$$\beta = \alpha_1 k_0^2 / g(1-s).$$

The dispersion curves obtained when $\beta < 1/4$; $1/4 < \beta < (1+s)^2/4$; $\beta > (1+s)^2/4$ are shown in Fig. 2, a-c, respectively. In these figures

$$\tilde{k}_{1,4} = \frac{(1+s) \mp \sqrt{(1+s)^2 - 4\beta}}{2\beta}, \quad \beta \leqslant \frac{(1+s)^2}{4};$$
$$\tilde{k}_{2,3} = \frac{1 \mp \sqrt{1-4\beta}}{2\beta}, \quad \beta \leqslant \frac{1}{4}.$$

The sections of the curves corresponding to waves with negative energy are hatched.

Let us determine the real values of β. Having set $\alpha_0 = 5.10^{-5}$ m³/sec² and s = 0.8, we obtain

$$\beta = 4 \cdot 10^{-4} / u_0^4,$$

where $u_0$ is measured in m/sec. If 0.2 m/sec $\leq u_0 \leq$ 1 m/sec, then $4 \cdot 10^{-4} \leq \beta \leq 1/4$.

At $\beta \approx 4 \cdot 10^{-4}$ $\lambda_1 = 2$ m; $\lambda_2 = 2 \cdot 10^{-4}$ m; where $\beta \approx 1/4$ $\lambda_1 = 6 \cdot 10^{-3}$ m; $\lambda_4 = 3 \cdot 10^{-4}$ m, m/sec, $\lambda_i = 2\pi/\tilde{k}_i k_0$.

Now let us consider the case of low viscosity (low value of ε). First we will examine the region of purely capillary waves ($\lambda < 10^{-3}$ m). For such waves, $\omega_0^2 = \alpha_0 k^3$, E = N. Considering that

$$N \sim 1, \ \frac{l}{k} = \sqrt{\frac{\tilde{\omega}}{\varepsilon}}\,(1-i),$$

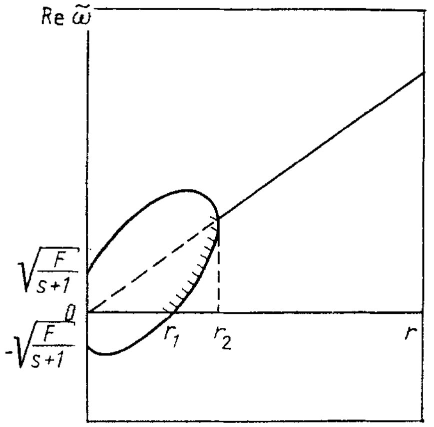

Fig. 1. Dependence of dimensionless frequency $\tilde{\omega}$ on r.

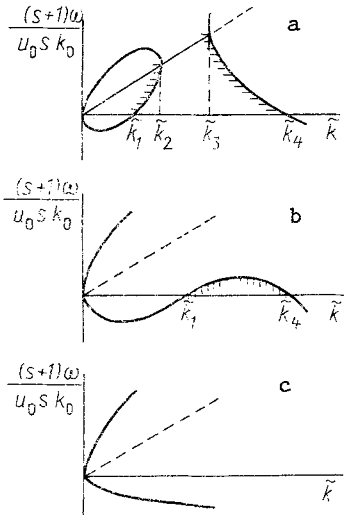

Fig. 2. Dependence of frequency ω on the dimensionless wave number $\tilde{k}$.

to within $\varepsilon^{0.5}$ we obtain the following from (6)

$$Z_2 \simeq Z_0 + \frac{\tilde{\omega}^2}{2}\sqrt{\frac{\varepsilon}{\tilde{\omega}}}(1+i) = 0.$$

It is easily found from this that the imaginary complement to the frequency $\tilde{\omega}$ is determined by the expression

$$\omega' = -\frac{\tilde{\omega}^2}{2}\sqrt{\frac{\varepsilon}{\tilde{\omega}}}\left(\frac{\partial Z_0}{\partial \tilde{\omega}}\right)^{-1}.$$

For waves with negative energy, $\omega' > 0$. Such a situation leads to instability.

For a pure surface, N = 0 and $\tilde{\omega}' = \tilde{\omega}'_0 = -(2\varepsilon\tilde{\omega})/(\partial Z_0/\partial\tilde{\omega})$. Thus, for waves with negaive energy we again obtain $\tilde{\omega}'_0 > 0$. The ratio of the amplification factors is

$$\frac{\tilde{\omega}'}{\tilde{\omega}'_0} \sim \varepsilon^{-0.5} \gg 1.$$

Thus, the presence of the surfactant leads to amplification of waves with negative energy. Meanwhile, the amplification factor on a surface covered by a surfactant turns out to be considerably larger than the factor on a pure surface.

In the long-capillary-wave approximation, $\omega_0 \simeq gk$. At $k \sim 10^2\ \mathrm{m}^{-1}$, we can assume that $\varepsilon \ll \varepsilon\frac{l}{k} \ll E \ll E\frac{l}{k}$.

To within infinitesimals of order E, we find from (5) that

$$\tilde{\omega}' = -\tilde{\omega}^2 \sqrt{\frac{\tilde{\omega}}{\varepsilon}}\, E^2 \left(\frac{\partial Z_0}{\partial \tilde{\omega}}\right)^{-1}.$$

The ratio of the amplification factors is

$$\frac{\tilde{\omega}'}{\tilde{\omega}_0'} \sim E^2 \varepsilon^{-1,5} \gg 1.$$

Thus, the surfactant also exerts a substantial amplifying effect in the region of long capillary waves with negative energy. Similar results can be obtained for m = 1.

It should be noted that in the case of waves with positive energy, the effect of the surfactant leads to attenuation of capillary waves [1, 2].

## CONCLUSION

It was concluded above that the presence of surface-active agents leads to amplification of long capillary waves with negative energy. This conclusion can be put to broad use in controlling the transport of multicomponent media, since capillary waves [1, 2] assist in the formation and buildup of gravitational waves. The latter in turn lead to undesirable pulsations of fluid pressure and velocity. Surface-active agents can be present in transported media in the form of impurities, but sometimes they must be specially introduced (as corrosion inhibitors, for example) [9]. As has already been noted, the manifestation of the Marangoni effect might also be connected with the presence of surface-active ions which alter the electrical component of surface tension. Thus, under certain conditions, the effect of surface-active ions on the attenuation or intensification of waves may be just as important. This fact might in particular explain some of the phenomena associated with the effect of weak electromagnetic fields on the flow of multicomponent media.

## NOTATION

$\rho_j$, $\mu_j$, and $\nu_j$, density, viscosity, and kinematic viscosity of the j-th layer (j = 1 and j = 2 correspond to the upper and lower layers); $c = \omega/k$, phase velocity; $\omega$, frequency; k, wave number; $u_{0j}(z)$, velocities of the fluids in the undisturbed state; $u_j$ and $v_j$, perturbations of velocities along the x and z axes, respectively; $\sigma(\Gamma)$, surface tension; $p_j$, pressure; g, acceleration due to gravity; $\tilde{\omega} = \omega/\omega_0$; $\omega_0^2 = (1-s)gk + \alpha_0 k^3$; $s = \rho_1/\rho_2$; $\alpha_0 = \sigma_0/\rho_2$; $c_0 = \omega_0/k$; $\varepsilon = 2\nu_2 k^2/\omega_0$; $E = \alpha_0 N k^3/\omega_0^3$; $N = \left(\frac{\partial\sigma}{\partial\Gamma}\right)\frac{\Gamma_0}{\sigma_0}$; $p_\sigma = -\sigma\frac{\partial^2\eta}{\partial x^2}$; $p_\tau = \frac{\partial\sigma}{\partial x}$.

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